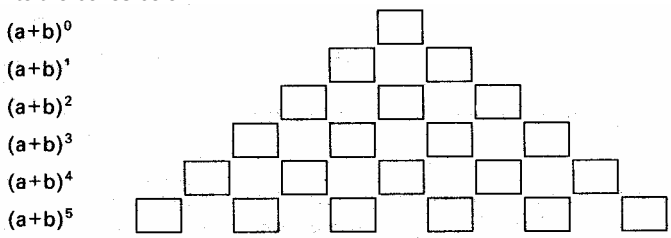
UNIT 2: Binomial Theorem for Positive Integral Indices

Specific Objectives:

1. To recognize the notations n! and C_r^n .
2. To learn to expand a binomial with positive integral index by the binomial theorem.

Detailed Content	Time Ratio	Notes on Teaching
2.1 The n! and C_r^n notations	1	The definitions of n! and C_r^n should be introduced. The idea of permutation and combination may only be mentioned to abler students. The definition of O! = 1 should also be mentioned. It is desirable to bring students' attention to the other forms of C_r^n , viz, ${}_nC_r$ and $\binom{n}{r}$. Students are expected to be able to verify $C_r^n = C_{n-r}^n$ and $C_{r-1}^n + C_r^n = C_r^{n+1}$. In the latter part, teachers should guide students to start from L.H.S. and end up with R.H.S. Examples like the one below can be given. *Example* Solve for n if $C_{n+1}^{18} = C_{2n+1}^{18}$.
2.2 The Pascal Triangle	1	Teachers can ask students to expand $(a + b)^2$, $(a + b)^3$, $(a + b)^4$ and $(a + b)^5$ by multiplication and fill the coefficients of the terms of the expanded expressions into the boxes below. 

Detailed Content	Time Ratio	Notes on Teaching
2.3 Expanding Binomials Using the Pascal Triangle	2	Teachers should guide students to discover that the coefficients can be expressed as C_r^n and the properties of the elements of the Pascal triangle such as $C_r^n = C_{n-r}^n$, $C_0^n = 1$, $C_n^n = 1$ and $C_{n-r}^n + C_r^n = C_r^{n+1}$. Students are expected to expand $(a + b)^n$ with the aid of Pascal triangle up to and including n=5. The following examples are relevant. *Example 1* Expand (*a*) $(2x+3)^4$ in descending powers of x, (*b*) $(3x^2-1)^5$ in ascending powers of x. *Example 2* Find the term independent of x in the expansion of $(2x^3 - \frac{1}{3x^2})^5$.
2.4 Binomial Theorem for Positive Integral Indices	4	Starting with the Pascal triangle, teachers can introduce binomial theorem for positive integral indices. The proof of the theorem using mathematical induction is a good exercise for the students. Students should discover for themselves that in expanding $(x + y)^n$, (*a*) there are (n+1) terms and (*b*) the $(r + 1)^{th}$ term is $C_r^n x^{n-r}y^r$, if the expansion is expressed in descending powers of x. The determination of the greatest term and relations between coefficients are not necessary. Examples such as the following can be given.

Detailed Content	Time Ratio	Notes on Teaching
		Example 1 Expand (*a*) $(2x+3y)^4$ (*b*) $(3x-\frac{2}{x})^5$. *Example 2* Find the coefficient of x^3 in the expansion of $(3-\frac{1}{2}x)^6\cdot(1+x)^5$. *Example 3* In the expansion of $(1+ax)(1+bx)^6$, the coefficients of x and x^2 are respectively 0 and $-\frac{21}{4}$, find the values of a and b. Although problems of multinomial expansion can be solved by repeated binomial expansions, teachers should avoid teaching multinomial expansion beyond the trinomials. The following can be useful exercises. *Example 4* Expand $(1-2x+3x^2)^3$ in ascending powers of x. *Example 5* Find the constant term in the expansion of $(1+\frac{1}{2x}-3x)^4$. For abler students, teachers may mention that the binomial theorem is also true for negative integral indices. Nevertheless, further development should be avoided.
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